The Combinatorial Structure of Trigonometry

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Abstract
The native mathematical language of trigonometry is combinatorial. Two interrelated combinatorial symmetric functions underlie trigonometry. We use their characteristics to derive identities for the trigonometric functions of multiple distinct angles. When applied to the sum of an infinite number of infinitesimal angles, these identities lead to the power series expansions of the trigonometric functions. When applied to the interior angles of a polygon they lead to two general constraints satisfied by the corresponding tangents. In the case of multiple equal angles they reduce to the Bernoulli identities. For the case of two distinct angles they reduce to the Ptolemy identity. They can also be used to derive the De Moivre-Cotes identity. The above results combined provide an appropriate mathematical combinatorial language and formalism for trigonometry and more generally polygonometry. This latter is the structural language of molecular organization, and is omnipresent in the natural phenomena of molecular physics, chemistry and biology. Polygonometry is as important in the study of moderately complex structures, as trigonometry has historically been in the study of simple structures.